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Mechanical Waves

① Velocity of waves: Velocity of longitudinal wave in liquid & gas

$$V = \sqrt{\frac{B}{\rho}}$$

where B is the bulk modulus of elasticity & ρ be the density

② Velocity of longitudinal wave in solid rod

$$V = \sqrt{\frac{Y}{\rho}}$$

where Y is young's modulus & ρ - density.

③ Velocity of transverse wave in a stretched string

$$V = \sqrt{\frac{T}{\mu}}$$

where, T is tension on the string
μ is mass per unit length

④ Velocity of electromagnetic waves

$$V = \frac{1}{\sqrt{\mu\varepsilon}}$$

where, μ is permeability & ε is permittivity

Velocity of sound in gas

Dimensional Method: Velocity of sound wave depends on the elasticity of the medium & its density.
Let V is velocity of sound
E is the Modulus of elasticity

ρ = Density of the medium,

Then

$$v \propto E^{x} \rho^{y}$$

$$v = k E^{x} \rho^{y} \quad \text{(i)}$$

where k is a proportionality const. x & y are the indices to be determined

Dimension of velocity $V = [LT^{-1}]$

Dimension of modulus of elasticity $E = [ML^{-1}T^{-2}]$

Dimension of density $\rho = [ML^{-3}]$

From (i) we get ~~[LT^{-1}]~~

$$[LT^{-1}] = [ML^{-1}T^{-2}]^{x} [ML^{-3}]^{y}$$

$$= [M]^{x+y} [L]^{-x-3y} [T]^{-2x}$$

Equating the indices of identical terms, we get

$$\boxed{E = \frac{\rho}{\Delta}}$$

$$x + y = 0 \quad \text{(ii)}$$

$$-x - 3y = 1 \quad \text{(iii)}$$

$$-2x = -1 \quad \text{(iv)}$$

$$+2x = +1$$

$$x = \frac{1}{2}$$

From (ii)

$$\frac{1}{2} + y = 0$$

$$y = -\frac{1}{2}$$

Putting these value in eqn (i)

$$v = k E^{\frac{1}{2}} \rho^{-\frac{1}{2}}$$

N	L
slow, T = const	Rapid T ≠ const
Isothermal	$dQ = 0$ Adiabatic

$$V = k\sqrt{\frac{E}{\rho}}$$

From exp $k = 1$

$$V = \sqrt{\frac{E}{\rho}}$$

Newton's formula for velocity of sound

When sound waves propagate through air, the compression & rarefraction are formed which process occur so slowly that the heat produced during the compression is given to the surrounding air while the heat loss during the rarefraction is gained from the surrounding air. So, the tempr of the medium remains same. Hence acc to Newton the propagation of sound through air is an isothermal process.

Eqn of isothermal process is

$$PV = \text{const.}$$

diff both side

$$d(PV) = d(\text{const})$$

$$PdV + VdP = 0$$

$$PdV = -VdP$$

$$P = \frac{-VdP}{dV}$$

$$P = \frac{-dP}{\frac{dV}{V}}$$

$$P = B$$

where $B = \frac{-dP}{\frac{dV}{V}}$ is the bulk modulus of air.

Now, Velocity of sound in air.

$$V = \sqrt{\frac{P}{\rho}}$$

$$V = \sqrt{\frac{P}{\rho}}$$

This formula is known as Newton's formula for veloc of sound.

Since,

Atms pressure $(P) = 1.01 \times 10^5 \, N/m^2$

Density of air at NTP $(\rho) = 1.29 \, kg/m^3$

Velocity of sound

$$V = \sqrt{\frac{1.01 \times 10^5}{1.29}}$$

280m/s

however the velocity of sound in air, the NTP is found experimentally as 323m/s which is quite greater than above calculated value. Such diff in values shows that there are discrepancies in Newton's assumptions & needs correction.

Laplace's correction:

→ Acc to laplace's the processes of compression & rarefraction occur so rapidly that neither heat is transfered to the surroundings during compression nor heat is taken from the surroundings during rarefraction. Thus the temp doesn't remain constant. As there is no exchange of heat so the sound waves in a gas propagate thru and adiabatic process.

Eqn of adiabatic process

$$PV^{\gamma} = \text{constant}$$

Differentiating both sides,

$$\left[\gamma = \frac{C_P}{C_V}\right]$$

ratio of two specific heats of a gas

$$dPV^{\gamma} = d\,\text{const}$$

$$P\,dV^{\gamma} + V^{\gamma}\,dP = 0$$

$$P \times \gamma \times V^{\gamma-1} dV + V^{\gamma}\,dP = 0$$

Dividing both sides by $V^{\gamma-1}$

$$P\gamma\,dV + V\,dP = 0$$

$$-\frac{dP}{dV}$$

$$\cancel{-V\,dP = P\gamma\,dV}$$

$$\cancel{-\frac{dP}{dV} = \frac{P\gamma}{V}}$$

$$\gamma P\,dV = -V\,dP$$

$$\gamma P = -V\frac{dP}{dV}$$

$$\gamma P = \frac{-\frac{dP}{dV}}{V}$$

$$\gamma P = \beta$$

velocity of sound in gas,

$$V = \sqrt{\frac{P}{\rho}}$$

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

This formula is known as Laplace's formula

At NTP,

$P = 1.01 \times 10^5 \ N/m^2$

For air, $\rho = 1.29 \ kg/m^3$

For air, $\gamma = 1.4$

$$V = \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.29}}$$

$$V = 331.2 \ m/s$$

This result closely agree's with the experimental value hence Laplace's formula gives the correct value of velocity of sound in air at NTP.

Factors affecting Velocity of sound

1) Effect of Tempr,

velocity of sound in air is,

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{--- (i)}$$

For 1 mole of gas, Eqn of state

$$PV = RT$$

$$\frac{PM}{\rho} = RT$$

$$\frac{P}{\rho} = \frac{RT}{M} \quad \text{--- (ii)}$$

If V_1 & V_2 are velocity of sound at tempr T_1 & T_2 resp then,

$$\frac{V_1}{V_2} = \sqrt{\frac{t_1}{t_2}}$$

From (i) & (ii)

$$V = \sqrt{\frac{\gamma RT}{M}}$$

For given gas, γ, R & M constant then

$$V \propto \sqrt{T}$$

Thus, velocity of sound in air is directly propr to the sqr root of tempr.

(ii) Effect of pressure: Velocity of sound in air is given by

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{(i)}$$

For 1 mole of gas, Eqn of state

$$PV = RT$$

$$\frac{PM}{\rho} = RT$$

$$\frac{P}{\rho} = \frac{RT}{M} \quad \text{(ii)}$$

From (i) & (ii)

$$V = \sqrt{\frac{\gamma RT}{M}}$$

At constant tempr & for gives gas γ, R, T, & M constant. Then

$$V = \text{constant}.$$

Since velocity of sound is independent of pressure at constant tempr, so the pressure does not affect velocity.

(iii) Effect of density,

Velocity of sound in air is given by,

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

At constant pressure,

$$V \propto \frac{1}{\sqrt{\rho}}$$

Thus velocity of sound in air is inversely prop to sq root of density.

Consider two gases of densities ρ_1 & ρ_2 at constant pressure velocity of sound is given

$$\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

(iv) Effect of humidity

→ Since the density of moist air is less than density of dry air. As velocity of sound in air is inversely prop to sq root of density. So, velocity of sound is more in humid air

$$V \propto \frac{1}{\rho}$$

$$V \propto \text{humidity}$$

#

$$V = \sqrt{\frac{E}{\rho}}$$

In solid $V = \sqrt{\frac{E}{\rho}}$

$E \to Y$

$V = \sqrt{\frac{Y}{\rho}}$ $Y \to$ Young modulus of elasticity

ρ - density of solid

In liquid (Fluid) $V = \sqrt{\frac{B}{\rho}}$

$B \to$ Bulk modulus of elasticity

In air,

$V = \sqrt{\frac{\gamma P}{\rho}}$ P - Atm pressure

$\gamma \to$ Molar heat capacity at const pressure to constant val. ρ - density

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→ Velocity of sound is high in solid because it travels 500m/s in solid so its high

v → high] than liquid &
ρ → high] air

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